Math 43 Midterm 3 Review Answers

In addition to the following review questions, you must be able to solve any of the questions from the 3D Lines & Planes handout.

[1] [a] $< -2, -4 > \cdot < x, y > = 0 \quad \Rightarrow \quad -2x - 4y = 0$

Let $x = 2$ and $y = -1$

$\dfrac{1}{\|< 2, -1 >\|} < 2, -1 > \;=\; \frac{1}{\sqrt{5}} < 2, -1 > \;=\; < \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} >$

[b] $-2 < 0 \quad \Rightarrow \quad \theta = \pi + \tan^{-1} \frac{-4}{-2} \approx 4.25$ radians

[c] $2 < 8\cos\frac{2\pi}{3}, 8\sin\frac{2\pi}{3} > - < -2, -4 > \;=\; 2 < -4, 4\sqrt{3} > - < -2, -4 > \;=\; < -8, 8\sqrt{3} > - < -2, -4 >$

$= \; < -6, 4 + 8\sqrt{3} > \;=\; -6\vec{i} + (4 + 8\sqrt{3})\vec{j}$

[2] [a] $\cos^{-1} \dfrac{< 0, 2, -3 > \cdot < -1, -3, 4 >}{\|< 0, 2, -3 >\| \, \|< -1, -3, 4 >\|} \;=\; \cos^{-1} \dfrac{-18}{\sqrt{13}\sqrt{26}} \;\approx\; 2.94$ radians

[b] $< 0, 2, -3 > \times < -1, -3, 4 > = < -1, 3, 2 >$

$\dfrac{1}{\|< -1, 3, 2 >\|} < -1, 3, 2 > \;=\; \frac{1}{\sqrt{14}} < -1, 3, 2 > \;=\; < -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}} >$

[c] $proj_{< -1, -3, 4 >} < 0, 2, -3 > \;=\; \dfrac{< 0, 2, -3 > \cdot < -1, -3, 4 >}{< -1, -3, 4 > \cdot < -1, -3, 4 >} < -1, -3, 4 > \;=\; \dfrac{-18}{26} < -1, -3, 4 >$

$= \; -\frac{9}{13} < -1, -3, 4 > \;=\; < \frac{9}{13}, \frac{27}{13}, -\frac{36}{13} >$

$< 0, 2, -3 > - < \frac{9}{13}, \frac{27}{13}, -\frac{36}{13} > \;=\; < -\frac{9}{13}, -\frac{1}{13}, -\frac{3}{13} >$

$< 0, 2, -3 > \;=\; < \frac{9}{13}, \frac{27}{13}, -\frac{36}{13} > + < -\frac{9}{13}, -\frac{1}{13}, -\frac{3}{13} >$

[d] $< -7 - x, 4 - y, -8 - z > = < -1, -3, 4 >$

$-7 - x = -1, \quad 4 - y = -3, \quad -8 - z = 4 \quad \Rightarrow \quad (x, y, z) = (-6, 7, -12)$

[e] $< a, b, -5 > = m < -1, -3, 4 > \quad \Rightarrow \quad a = -m, \quad b = -3m, \quad -5 = 4m \quad \Rightarrow \quad m = -\frac{5}{4}, \quad a = \frac{5}{4}, \quad b = \frac{15}{4}$

[f] $< 7, c, -5 > \cdot < -1, -3, 4 > = 0 \quad \Rightarrow \quad -7 - 3c - 20 = 0 \quad \Rightarrow \quad c = -9$

[3] [a] octant $2 + 4 = 6$

[b] $(-5 - 4, -2 + 6, 3 - 2) = (-9, 4, 1)$

[c] $< 3 - -3, 2 - 4, -1 - -2 > = < 6, -2, 1 >$

[d] $< -3 - -5, 4 - -2, -2 - 3 > = < 2, 6, -5 > = 2\vec{i} + 6\vec{j} - 5\vec{k}$

[e] $\sqrt{4 + 36 + 25} = \sqrt{65}$

[f] $-\frac{1}{\sqrt{65}} < 2, 6, -5 > = < -\frac{2}{\sqrt{65}}, -\frac{6}{\sqrt{65}}, \frac{5}{\sqrt{65}} >$

[g] $\frac{6}{\sqrt{41}} < 6, -2, 1 > = < \frac{36}{\sqrt{41}}, -\frac{12}{\sqrt{41}}, \frac{6}{\sqrt{41}} >$

[h] $(\sqrt{41})(3)\cos 2 \approx -7.99$

[i] $(\sqrt{41})(3)\sin 2 \approx 17.47$

[j] $\overrightarrow{RQ} \times \overrightarrow{RP} = < 6, -2, 1 > \times < -2, -6, 5 > = < -4, -32, -40 > = -4 < 1, 8, 10 >$

$\frac{1}{2} \| -4 < 1, 8, 10 > \| = 2 \|< 1, 8, 10 >\| = 2\sqrt{165}$

[k] $\cos^{-1} \dfrac{< 6, -2, 1 > \cdot < -2, -6, 5 >}{\|< 6, -2, 1 >\| \, \|< -2, -6, 5 >\|} \;=\; \cos^{-1} \dfrac{5}{\sqrt{41}\sqrt{65}} \;\approx\; 1.47$ radians

[l] $< 4, 0, -5 > \cdot < -5 - 3, -2 - 2, 3 - -1 > = < 4, 0, -5 > \cdot < -8, -4, 4 > = -52$

[m] normal vector $= \overrightarrow{RQ} \times \overrightarrow{RP} = < -4, -32, -40 >$ or $< 1, 8, 10 >$

$1(x + 5) + 8(y + 2) + 10(z - 3) = 0 \quad \Rightarrow \quad x + 8y + 10z - 9 = 0$

[n] $x = -5 + 6t, \quad y = -2 - 2t, \quad z = 3 + t$

[o] direction vector $= < -2, -3, 1 >$ or $< 2, 3, -1 >$

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z+1}{-1} \text{ or } -z-1$$

[p] center $= (\frac{-5+3}{2}, \frac{-2+2}{2}, \frac{3+-1}{2}) = (-1, 0, 1)$

radius $^2 = (3--1)^2 + (2-0)^2 + (-1-1)^2 = 24$

$(x+1)^2 + y^2 + (z-1)^2 = 24$

[4] $x < 0, y > 0, z > 0 \Rightarrow$ octant 2

$x < 0, y < 0, z > 0 \Rightarrow$ octant 3

$x > 0, y > 0, z < 0 \Rightarrow$ octant $1+4=5$

$x > 0, y < 0, z < 0 \Rightarrow$ octant $4+4=8$

[5] [a] $x^2 - 4x + y^2 + 6y + z^2 + 10z = -29$

$(x^2 - 4x + 4) + (y^2 + 6y + 9) + (z^2 + 10z + 25) = -29 + 4 + 9 + 25$

$(x-2)^2 + (y+3)^2 + (z+5)^2 = 9$

center $= (2, -3, -5)$

radius $= 3$

[b] $(x-2)^2 + (y+3)^2 + (0+5)^2 = 9 \Rightarrow (x-2)^2 + (y+3)^2 = -16$

no $xy-$trace

$(x-2)^2 + (0+3)^2 + (z+5)^2 = 9 \Rightarrow (x-2)^2 + (z+5)^2 = 0$

$xz-$trace is point $(2, 0, -5)$

$(0-2)^2 + (y+3)^2 + (z+5)^2 = 9 \Rightarrow (y+3)^2 + (z+5)^2 = 5$

$yz-$trace is circle in $yz-$plane, center $= (0, -3, -5)$, radius $= \sqrt{5}$